Pearson Edexcel

Mark Scheme (Results)

January 2022

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In Further Pure Mathematics (4PM1)

Paper 2R

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme - not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, a senior examiner must be consulted before a mark is given.
- Crossed out work should be marked **unless** the candidate has replaced it with an alternative response.

- **Types of mark**
    - M marks: method marks
    - A marks: accuracy marks
    - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
    - cao – correct answer only
    - ft – follow through
    - isw – ignore subsequent working
    - SC - special case
    - oe – or equivalent (and appropriate)
    - dep – dependent
    - indep – independent
    - awrt – answer which rounds to
    - eeoo – each error or omission

- **No working**
  If no working is shown then correct answers normally score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**
  If the final answer is wrong, always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.
  If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review.
  If there is a choice of methods shown, then award the lowest mark, unless the answer on the answer line makes clear the method that has been used.
  If there is no answer achieved then check the working for any marks appropriate from the mark scheme.

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.
  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.
  Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$$\left(x^2+bx+c\right)=\left(x+p\right)\left(x+q\right), \text{ where } |pq|=|c| \quad \text{leading to } x=\ldots.$$

$$\left(ax^2+bx+c\right)=\left(mx+p\right)\left(nx+q\right) \text{ where } |pq|=|c| \text{ and } |mn|=|a| \quad \text{leading to } x=\ldots.$$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=\ldots.$

3. Completing the square:

$$x^2+bx+c=0: \ \left(x\pm\tfrac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0 \qquad \text{leading to } x=\ldots.$$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $\left(x^n \to x^{n-1}\right)$

2. Integration:

Power of at least one term increased by 1. $\left(x^n \to x^{n+1}\right)$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is <u>not</u> quoted, the method mark can be gained by implication

from the substitution of <u>correct</u> values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers <u>may</u> be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

Question number	Scheme	Marks
1 (a)	$x > 3$	B1 (1)
(b)	$(2x-1)(x-5)$	M1
	Critical values are $x = \frac{1}{2}$ and $x = 5$	A1
	$\frac{1}{2} < x < 5$	B1ft (3)
(c)	$3 < x < 5$	B1ft (1)
		Total 5 marks

Part	Mark	Notes
(a)	**B1**	For $x > 3$
(b)	**M1**	For solving the given 3TQ by any method. See General Guidance.
	A1	For the correct values; $x = \frac{1}{2}$ and $x = 5$
	B1ft	For $\frac{1}{2} < x < 5$ ft their solutions to the 3TQ.
(c)	**B1ft**	For $3 < x < 5$ ft from parts (a) and (b)

Question number	Scheme	Marks
2 (a)	Gradient $=\dfrac{4+1}{3+7}=\dfrac{1}{2}$	M1
	$y+1=\dfrac{1}{2}(x+7)$	M1
	$x-2y+5=0$	A1 (3)
(b)	$AB=\sqrt{10^2+5^2}=5\sqrt{5}$ $AC=\sqrt{(-7--3)^2+(7--1)^2}=4\sqrt{5}$	M1
	$k=\dfrac{5}{4}$	A1 (2)
(c)	$-2=\dfrac{7-p}{-3-3}$	M1
	$12=7-p$	dM1
	$p=-5$	A1 (3)
		Total 8 marks

Part	Mark	Notes
(a)	**M1**	For finding the gradient of *AB*
	M1	For a fully correct method for finding the equation of a straight line. If $y=mx+c$ is used, then they must find a value for *c* for the award of this mark.
	A1	For $x-2y+5=0$ in the required form. Accept the terms in any order provided they are all on one side of the equation with the other = 0 e.g., $-2y+x+5=0$ or $2y-x-5=0$ etc
(b)	**M1**	For using Pythagoras to find both *AB* **and** *AC* $AB=\sqrt{(7--3)^2+(4--1)^2}=5\sqrt{5}$ and $AC=\sqrt{4^2+8^2}=4\sqrt{5}$
	A1	For $k=\dfrac{5}{4}$
(c)	**M1**	Obtains an equation using the perpendicular of their gradient from (a) and the point (3, *p*) $-2=\dfrac{7-p}{-3-3}$
	dM1	For a linear equation in *p* $12=7-p$
	A1	For $p=-5$
	ALT	
	M1	Finds the equation of the perpendicular using their gradient form part (a) $y-7=-2(x+3)\Rightarrow y=-2x+1$
	dM1	Substitutes *x* = 3 into their equation of the perpendicular to find a value for *y*
	A1	For $p=-5$

Question number	Scheme	Marks
3 (a)	$\dfrac{dy}{dx} = 2e^{2x}\sqrt{5x-3} + \dfrac{5e^{2x}}{2\sqrt{5x-3}}$	M1 A1 A1 (3)
(b)	$\dfrac{dy}{dx} = \dfrac{3x^2\cos 3x - x^3(-3\sin 3x)}{(\cos 3x)^2}$	M1 A1 A1 (3)
		Total 6 marks

Part	Mark	Notes
(a)	**M1**	For use of the product rule. Sum of two terms (either way round) There must be an attempt to differentiate both terms. See below. $(5x-3)^{\frac{1}{2}} \Rightarrow \frac{1}{2}\times k\times(5x-3)^{-\frac{1}{2}} \quad k \neq 0$ $e^{2x} \Rightarrow le^{2x} \quad l \neq 0$
	A1	For either term correct $2e^{2x}\sqrt{5x-3}$ or $\dfrac{5e^{2x}}{2\sqrt{5x-3}}$
	A1	For the correct derivative. $\dfrac{dy}{dx} = 2e^{2x}\sqrt{5x-3} + \dfrac{5e^{2x}}{2\sqrt{5x-3}}$ Or $\dfrac{dy}{dx} = 2e^{2x}(5x-3)^{\frac{1}{2}} + \dfrac{5}{2}e^{2x}(5x-3)^{-\frac{1}{2}}$ oe
(b)	**M1**	For an attempt at the use of the quotient rule. - There must be an acceptable attempt to differentiate both terms $x^3 \Rightarrow 3x^2$ $\cos(3x) \Rightarrow -m\sin 3x \quad m \neq 0$ - The denominator must be squared. - The terms in the numerator must be subtracted in either order.
	A1	For one term correct $3x^2\cos 3x$ or $x^3(-3\sin 3x)$
	A1	For the fully correct derivative.

Question number	Scheme	Marks
4 (a)	$\alpha+\beta=-2 \qquad \alpha\beta=\frac{3}{2}$	B1
	$(\alpha+\beta)^2-2\alpha\beta=\alpha^2+\beta^2$	M1
	$(-2)^2-2\left(\frac{3}{2}\right)=4-3=1$ *	M1 A1cso (4)
ALT		
	$2\alpha^2+4\alpha+3=0$ and $2\beta^2+4\beta+3=0$	{B1}
	$2\ \ \alpha^2+\beta^2\ \ +4\ \ \alpha+\beta\ \ +6=0$	{M1}
	$\alpha^2+\beta^2\ \ =-2\ \ -2\ \ -3=1$*	{M1} {A1cso} (4)
(b)	$\alpha^4+\beta^4=(\alpha^2+\beta^2)(\alpha^2+\beta^2)-2\alpha^2\beta^2$	M1
	$1\times1-2\left(\frac{3}{2}\right)^2=-\frac{7}{2}$	M1 A1 (3)
ALT		
	$\alpha^2+\beta^2\ \ ^2=1=\alpha^4+2\alpha^2\beta^2+\beta^4=\alpha^4+\beta^4+2\times\frac{9}{4}$	{M1}
	$\alpha^4+\beta^4=1-\frac{9}{2}=-\frac{7}{2}$	{M1} {A1} (3)
(c)	Product of the roots: $\alpha^4\beta^4=\left(\frac{3}{2}\right)^4=\frac{81}{16}$	M1
	(Sum of the roots: $\alpha^4+\beta^4=-\frac{7}{2}$)	
	$16x^2+56x+81=0$	M1 A1ft (3)
		Total 10 marks

Part	Mark	Notes
(a)	B1	For $\alpha+\beta=-2$ and $\alpha\beta=\dfrac{3}{2}$
	M1	For $(\alpha+\beta)^2-2\alpha\beta=\alpha^2+\beta^2$ This must be correct
	M1	For substituting their sum and product into their expansion for $\alpha^2+\beta^2$ $(-2)^2-2\left(\dfrac{3}{2}\right)$
	A1cso	For obtaining the given expression $\alpha^2+\beta^2=1$
	ALT	
	B1	For $2\alpha^2+4\alpha+3=0$ and $2\beta^2+4\beta+3=0$
	M1	For $2\ \alpha^2+\beta^2\ +4\ \alpha+\beta\ +6=0$
	M1	For $-2\ -2\ -3$
	A1cso	For obtaining the given expression $\alpha^2+\beta^2=1$
(b)	M1	For $\alpha^4+\beta^4=(\alpha^2+\beta^2)(\alpha^2+\beta^2)-2\alpha^2\beta^2$ This must be correct
	M1	For substituting $\alpha^2+\beta^2=1$ and their product from (a) into their expansion for $\alpha^4+\beta^4$ $\alpha^4+\beta^4=1\times1-2\left(\dfrac{3}{2}\right)^2$
	A1	For the correct value $\alpha^4+\beta^4=-\dfrac{7}{2}$
	ALT	
	M1	For $1=\alpha^4+2\alpha^2\beta^2+\beta^4$
	M1	For $1=\alpha^4+\beta^4+2\times\dfrac{9}{4}$
	A1	For $\alpha^4+\beta^4=-\dfrac{7}{2}$
(d)	M1	For product of the roots: $\alpha^4\beta^4\left(=\left(\dfrac{3}{2}\right)^4=\dfrac{81}{16}\right)$
	M1	For use of x^2- (their sum) $x+$(their product)$=[0]$
	A1	For $16x^2+56x+81=0$ Must include = 0

Question number	Scheme	Marks
5 (a)	$\left[S_\infty =\right]\dfrac{12}{1-\dfrac{3}{8}}=\dfrac{96}{5}$	M1 A1 (2)
(b)	$ar^5 = 12\left(\dfrac{3}{8}\right)^5$	M1
	$=\dfrac{2^2\times 3\times 3^5}{2^{15}}=\dfrac{3^6}{2^{13}}$ *	M1 A1cso (3)
(c)	e.g. $u_n = 12\left(\dfrac{3}{8}\right)^{n-1}$	M1
	$\log_2 u_n = \log_2 12 + \log_2\left(\dfrac{3}{8}\right)^{n-1}$	M1
	$\log_2 u_n = \log_2 12 + (n-1)\left[\log_2 3 - \log_2 8\right]$	M1
	$\log_2 u_n = \log_2 3 + 2\log_2 2 + (n-1)[\log_2 3 - 3\log_2 2]$	M1
	$\log_2 u_n = n\log_2 3 - 3n + 5$ *	A1 cso (5)
		Total 10 marks

Part	Mark	Notes
(a)	M1	For the correct use of $\frac{a}{1-r}$
	A1	For the correct value $\frac{96}{5}$
(b)	M1	For the correct use of ar^{n-1}
	M1	For expressing 12 $\left(2^2\times 3\right)$ and 8 $\left(2^3\right)$ as powers of 2
	A1cso	For obtaining the given expression with no errors seen.
(c)	M1	For the correct use of ar^{n-1} and the given values of a and r to give $u_n = 12\left(\frac{3}{8}\right)^{n-1}$
	M1	For taking logs [base 2] of both sides and applying the addition law.
	M1	For applying the power law and subtraction laws to $\log_2\left(\frac{3}{8}\right)^{n-1}$ $\log_2\left(\frac{3}{8}\right)^{n-1} = (n-1)\left[\log_2 3 - \log_2 8\right]$
	M1	For obtaining $\log_2 12 = \log_2 3 + 2\log_2 2$
	A1cso	For obtaining the given equation with no errors seen.
	ALT	
	M1	For the correct use of ar^{n-1} and the given values of a and r to give $u_n = 12\left(\frac{3}{8}\right)^{n-1}$
	M1	For rearranging the equation to obtain: $U_n = 12\left(\frac{3}{8}\right)^{n-1} = 2^2\times 3\times\frac{3^{n-1}}{2^{3(n-1)}} = \frac{3^n}{2^{3n-5}}$
	M1	For taking logs of both sides and applying the addition (subtraction) law. $\log_2 U_n = \log_2 3^n - \log_2\left(3n-5\right)$
	M1	For applying the power law to obtain: $\log_2 U_n = n\log_2 3 - \log_2\left(3n-5\right)$
	A1 cso	For obtaining the given equation with no errors seen.

Question number	Scheme	Marks
6 (a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2}{\sqrt{x}}$	M1
	When $x=9$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2}{3}$	A1
	$y-12=\frac{2}{3}(x-9)$	M1
	When $y=0$ $-12=\frac{2}{3}(x-9)$	M1
	$x=-9$ So $(-9,0)$	A1 (5)
(b)	Gradient of Normal: $-\frac{3}{2}$	M1
	$y-12=-\frac{3}{2}(x-9)$	M1
	When $y=0$ $-12=-\frac{3}{2}(x-9)$	M1
	$x=17$ So $(17,0)$	A1 (4)
(c)	$\frac{1}{2}\times12\times26=156$	M1 A1 (2)
		Total 11 marks

Part	Mark	Notes
(a)	M1	For an attempt to differentiate y wrt x. Accept $\frac{dy}{dx} = 2x^{-\frac{1}{2}}$ Allow as a minimum $\frac{dy}{dx} = kx^{-\frac{1}{2}}$ where k is a constant $\neq 0$
	A1	For $\frac{dy}{dx} = \frac{2}{3}$
	M1	For a complete method to find the equation of a straight line. If they use $y = mx + c$ then they must reach a value for c for this mark. Accept the equation of the line with their value of $\frac{dy}{dx}$ using the given coordinates.
	M1	For substitution of $y = 0$ to find a value for x
	A1	For $(-9, 0)$
(b)	M1	For gradient of Normal: $-\frac{3}{2}$ Which is the negative reciprocal of their gradient obtained in (a)
	M1	For a complete method to find the equation of a straight line with a gradient of $-\frac{3}{2}$ (ft their gradient of normal) If they use $y = mx + c$ then they must reach a value for c for this mark.
	M1	For substitution of $y = 0$ to find a value for x
	A1	For $(17, 0)$
(c)	M1	For any correct method for finding the area of a triangle. e.g., $A = \frac{1}{2} \times 12 \times ('17' - '-9') = 156$ OR $A = \frac{1}{2}\begin{bmatrix} 9 & -9 & 17 & 9 \\ 12 & 0 & 0 & 12 \end{bmatrix} = \frac{1}{2}\left[(0+0+204)-(0+0-108)\right] = 156$
	A1	For 156

Question number	Scheme	Marks
7 (a)	$\dfrac{3+\left(1-\cos^2\theta\right)}{\cos\theta-2}=3\cos\theta$	M1
	$4-\cos^2\theta=3\cos^2\theta-6\cos\theta$	M1
	$(2\cos\theta+1)(2\cos\theta-4)=0$	M1
	($\cos\theta=2$ does not exist so) $\cos\theta=-\dfrac{1}{2}$ *	A1 cso (4)
(b)	$\cos 3x=-\dfrac{1}{2}$	M1
	$3x=120^\circ,\ 240^\circ,\ 480^\circ$	A1
	$x=40^\circ,\ 80^\circ,\ 160^\circ$	A1 A1 (4)
		Total 8 marks

Part	Mark	Notes
(a)	**M1**	For the correct use of $\sin^2\theta+\cos^2\theta=1$
	M1	For multiplying both sides by $\cos\theta-2$ and expanding brackets
	M1	For solving their 3TQ using any method. $4\cos^2\theta-6\cos\theta-4=0\Rightarrow 2\left(2\cos\theta-1\right)\left(\cos\theta-2\right)=0$ See General Guidance.
	A1cso	For obtaining the given equation: $\cos\theta=-\dfrac{1}{2}$ Must reject $\cos\theta=2$
	ALT	
	M1	For the correct use of $\sin^2\theta+\cos^2\theta=1$
	M1	Factorises LHS $\dfrac{4-\cos^2\theta}{\cos\theta-2}=3\cos\theta\Rightarrow\dfrac{\left(2-\cos\theta\right)\left(2+\cos\theta\right)}{\cos\theta-2}=3\cos\theta$
	M1	Cancels through by $\cos\theta-2$ and solves their linear equation in terms of $\cos\theta$ $-\left(2+\cos\theta\right)=3\cos\theta\Rightarrow\cos\theta=...$
	A1cso	For obtaining the given equation: $\cos\theta=-\dfrac{1}{2}$
(b)	**M1**	For $\cos 3x=-\dfrac{1}{2}$
	A1	For $3x=120^\circ$ or any other correct angle, e.g. even $3x=-120^\circ$ Allow an angle in radians for this mark. E.g. $3x=\dfrac{2\pi}{3}$
	A1	For one from $x=40^\circ,\ 80^\circ,\ 160^\circ$
	A1	For all angles correct with no additional angles within range. $x=40^\circ,\ 80^\circ,\ 160^\circ$ Ignore any angles out of range.

		Penalise any extra angles within range by deducting the final A mark.

Question number	Scheme	Marks
8 (a)	$4\times1.5=6$	B1
	$\pi r^2=6$	M1
	$r=\sqrt{\dfrac{6}{\pi}}$	A1 (3)
(b)	$\dfrac{\mathrm{d}A}{\mathrm{d}r}=2\pi r$	M1
	$\dfrac{\mathrm{d}r}{\mathrm{d}t}=\dfrac{\mathrm{d}r}{\mathrm{d}A}\times\dfrac{\mathrm{d}A}{\mathrm{d}t}\left[=\dfrac{1}{2\pi r}\times1.5\right]$	M1
	$=\dfrac{1}{2\pi\sqrt{\dfrac{6}{\pi}}}\times1.5$	M1
	$=0.173$	A1 (4)
		Total 7 marks

Part	Mark	Notes
(a)	**B1**	For $(4\times1.5=)6$
	M1	For $\pi r^2=6$ an attempt to rearrange to make r the subject
	A1	For $r=\sqrt{\dfrac{6}{\pi}}$
(b)	**M1**	For $\dfrac{\mathrm{d}A}{\mathrm{d}r}=2\pi r$ This must be correct.
	M1	For application of $\dfrac{\mathrm{d}r}{\mathrm{d}t}=\dfrac{\mathrm{d}r}{\mathrm{d}A}\times\dfrac{\mathrm{d}A}{\mathrm{d}t}$
	M1	For substituting their r into $\dfrac{\mathrm{d}r}{\mathrm{d}t}$
	A1	For awrt 0.173

Question number	Scheme	Marks
9 (a)	$\cos\alpha = \dfrac{3}{\sqrt{13}}$	B1 (1)
(b)	$h = \sqrt{17^2 - \left(9^2 + 12^2\right)} = 8$	M1 M1 A1 (3)
(c)	Let M be the midpoint of BC $\tan\theta^\circ = \dfrac{h}{OM} = \dfrac{8}{12} = \dfrac{2}{3}$ *	M1 A1 (2)
(d)	Let N be the midpoint of EM $EM = \sqrt{8^2 + 12^2} = 4\sqrt{13}$ $NO = \sqrt{12^2 + \left(2\sqrt{13}\right)^2 - 2(12)\left(2\sqrt{13}\right)\left(\dfrac{3}{\sqrt{13}}\right)} \Rightarrow NO = 2\sqrt{13}$ Hence triangle ONM is isosceles $180 - 2\times 33.7 = 112.6^\circ$	 M1 M1 M1 A1 (4)
		Total 10 marks

Part	Mark	Notes
(a)	**B1**	For $\cos\alpha = \dfrac{3}{\sqrt{13}}$
(b)	**M1**	For use of Pythagoras to OC find e.g. $\sqrt{\left(9^2 + 12^2\right)}$ oe
	M1	For use of Pythagoras to find h e.g. $\sqrt{17^2 - \left(9^2 + 12^2\right)}$
	A1	For 8
(c)	**M1**	For $\tan\theta^\circ = \dfrac{h}{OM} = \dfrac{8}{12}$ (condone the omission of the degree sign)
	A1 cso	For obtaining the given result
(d)	**M1**	For use of Pythagoras to find EM e.g. $\sqrt{8^2 + 12^2}$
	M1	For use of the cosine rule to find NO e.g. $\sqrt{12^2 + \left(2\sqrt{13}\right)^2 - 2(12)\left(2\sqrt{13}\right)\left(\dfrac{3}{\sqrt{13}}\right)}$
	M1	For $180 - 2\times 33.7$
	A1	For 112.6°

Question number	Scheme	Marks
10 (a)	$\sin x+1=\cos x+1 \Rightarrow \tan x=1$	M1
	$x=\dfrac{\pi}{4},\dfrac{5\pi}{4}$	A1 A1 (3)
(b)	Area R_1 :	
	$\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}}(\sin x+1)-(\cos x+1)\,\mathrm{d}x=\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}}(\sin x-\cos x)\,\mathrm{d}x$	M1
	$\left[-\cos x-\sin x\right]_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$	M1
	$\left(\dfrac{\sqrt{2}}{2}+\dfrac{\sqrt{2}}{2}\right)-\left(-\dfrac{\sqrt{2}}{2}-\dfrac{\sqrt{2}}{2}\right)=2\sqrt{2}$	M1 A1
	Area R_2 :	
	$\int_{\pi}^{\frac{5\pi}{4}}(\cos x+1)\,\mathrm{d}x+\int_{\frac{5\pi}{4}}^{\frac{3\pi}{2}}(\sin x+1)\,\mathrm{d}x$	M1
	$\left[\sin x+x\right]_{\pi}^{\frac{5\pi}{4}}+\left[-\cos x+x\right]_{\frac{5\pi}{4}}^{\frac{3\pi}{2}}$	A1
	$\left[\left(-\dfrac{\sqrt{2}}{2}+\dfrac{5\pi}{4}\right)-\pi\right]+\left[\dfrac{3\pi}{2}-\left(\dfrac{\sqrt{2}}{2}+\dfrac{5\pi}{4}\right)\right]=-\sqrt{2}+\dfrac{1}{2}\pi$	M1 A1
	area of R_1 : area of R_2 = $2:\left(\dfrac{\pi\sqrt{2}}{4}-1\right)$ oe	A1 (9)
		Total 12 marks

Part	Mark	Notes
(a)	**M1**	For $\tan x = 1$
	A1	For $x = \dfrac{\pi}{4}$ [Allow 45º]
	A1	For $x = \dfrac{5\pi}{4}$ [Allow 225º]
(b)	**M1**	For stating $A = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x + 1) - (\cos x + 1)\,\mathrm{d}x$
	M1	For attempting to integrate their expression for the area. This must be correct for this mark. $\int (\sin x - \cos x)\ \mathrm{d}x = -\cos x - \sin x$
	M1	For substitution of correct limits
	A1	For $2\sqrt{2}$
	M1	For attempting $\int \mathrm{f}(x)\mathrm{d}x + \int \mathrm{g}(x)\mathrm{d}x$ with the correct limits. $\int_{\pi}^{\frac{5\pi}{4}} (\cos x + 1)\mathrm{d}x + \int_{\frac{5\pi}{4}}^{\frac{3\pi}{2}} (\sin x + 1)\mathrm{d}x$
	M1	For attempting to integrate, which must be correct for this mark.
	M1	For substitution of their limits correctly.
	A1	For $-\sqrt{2} + \dfrac{1}{2}\pi$
	A1	For the simplified ratio $2 : \left(\dfrac{\pi\sqrt{2}}{4} - 1\right)$

Question	Scheme	Marks
11(a)	$\overrightarrow{AB} = -\mathbf{a}+\mathbf{b} \quad \overrightarrow{AC} = -\mathbf{a}+\frac{2}{3}\mathbf{b} \quad \overrightarrow{AM} = -\mathbf{a}+\frac{5}{6}\mathbf{b}$	B2
	$\overrightarrow{NB} = -\frac{1}{2}\overrightarrow{AC}+\overrightarrow{AB} = -\frac{1}{2}\left(-\mathbf{a}+\frac{2}{3}\mathbf{b}\right)+\left(-\mathbf{a}+\mathbf{b}\right) = \left[-\frac{\mathbf{a}}{2}+\frac{2\mathbf{b}}{3}\right]$ OR $\overrightarrow{NB} = -\frac{1}{2}\overrightarrow{AC}+\overrightarrow{AB} = \frac{1}{2}\left(-\mathbf{a}+\frac{2}{3}\mathbf{b}\right)+\frac{1}{3}\mathbf{b} = \left[-\frac{\mathbf{a}}{2}+\frac{2\mathbf{b}}{3}\right]$	M1
	Two statements for $\overrightarrow{OP}$ using two parameters $\overrightarrow{OP} = \overrightarrow{OA}+\lambda\overrightarrow{AM} = \mathbf{a}+\lambda\left(-\mathbf{a}+\frac{5}{6}\mathbf{b}\right) = \left[\mathbf{a}(1-\lambda)+\frac{5}{6}\lambda\mathbf{b}\right]$	M1
	$\overrightarrow{OP} = \overrightarrow{OB}-\mu\overrightarrow{NB} = \mathbf{b}-\mu\left(-\frac{\mathbf{a}}{2}+\frac{2\mathbf{b}}{3}\right) = \left[\mathbf{a}\left(\frac{\mu}{2}\right)+\mathbf{b}\left(1-\frac{2\mu}{3}\right)\right]$	M1
	Equates coefficients $\Rightarrow 1-\lambda = \frac{\mu}{2} \qquad \frac{5}{6}\lambda = 1-\frac{2\mu}{3}$	M1
	Solves S. E. $\Rightarrow \lambda = \frac{2}{3} \qquad \mu = \frac{2}{3}$	M1
	$\Rightarrow \overrightarrow{OP} = \mathbf{a}+\frac{2}{3}\left(-\mathbf{a}+\frac{5}{6}\mathbf{b}\right) = \frac{1}{3}\mathbf{a}+\frac{5}{9}\mathbf{b}$	M1A1
		[9]
(b)	Any two from $\overrightarrow{CP} = \frac{1}{3}\mathbf{a}-\frac{1}{9}\mathbf{b} \qquad \overrightarrow{PQ} = \frac{1}{6}\mathbf{a}-\frac{1}{18}\mathbf{b} \qquad \overrightarrow{CQ} = \frac{1}{2}\mathbf{a}-\frac{1}{6}\mathbf{b}$	B2
	$\overrightarrow{CQ} = \frac{3}{2}\overrightarrow{CP}$ OR $\overrightarrow{PQ} = 2\overrightarrow{CP}$ OR $\overrightarrow{CQ} = 3\overrightarrow{PQ}$	M1
	with a conclusion	A1 [4}
		Total 13 marks

Part	Mark	Notes
(a)	B1	For at least one from $\overrightarrow{AB}=-\mathbf{a}+\mathbf{b}$ $\quad\overrightarrow{AC}=-\mathbf{a}+\frac{2}{3}\mathbf{b}$ $\quad\overrightarrow{AM}=-\mathbf{a}+\frac{5}{6}\mathbf{b}$
	B1	All three correct.
	M1	For a correct vector statement for $\overrightarrow{NB}$
	There are several paths to $\overrightarrow{OP}$. Those below are examples. For the award of each of the two M marks, there must be two different paths using two distinct parameters.	
	M1	For either vector statement for $\overrightarrow{OP}$ using a parameter e.g. $\overrightarrow{OP}=\overrightarrow{OA}+\lambda\overrightarrow{AM}=\mathbf{a}+\lambda\left(-\mathbf{a}+\frac{5}{6}\mathbf{b}\right)=\left[\mathbf{a}(1-\lambda)+\frac{5}{6}\lambda\mathbf{b}\right]$
	M1	For a second vector statements for $\overrightarrow{OP}$ using a different parameter e.g. $\overrightarrow{OP}=\overrightarrow{OB}-\mu\overrightarrow{NB}=\mathbf{b}-\mu\left(-\frac{\mathbf{a}}{2}+\frac{2\mathbf{b}}{3}\right)=\left[\mathbf{a}\left(\frac{\mu}{2}\right)+\mathbf{b}\left(1-\frac{2\mu}{3}\right)\right]$
	M1	For equating coefficients of **a** and **b**
	dM1	For solving the simultaneous equations This mark is dependent on the previous 3 M marks $\left[\lambda=\frac{2}{3}\quad \text{OR} \quad \mu=\frac{2}{3}\right]$
	M1	For substituting their value of λ or μ into their $\overrightarrow{OP}=\overrightarrow{OB}-\mu\overrightarrow{NB}$ or $\overrightarrow{OP}=\overrightarrow{OA}+\lambda\overrightarrow{AM}$
	A1	For the correct vector $\overrightarrow{OP}=\frac{1}{3}\mathbf{a}+\frac{5}{9}\mathbf{b}$
	SC – If $\overrightarrow{CQ}$ is assumed to be a straight line and is used to find used $\overrightarrow{OP}$, award all the marks as above but withhold the final A mark.	
(b)	B2	For any two vectors from $\overrightarrow{CP}=\frac{1}{3}\mathbf{a}-\frac{1}{9}\mathbf{b}$ or $\overrightarrow{PQ}=\frac{1}{6}\mathbf{a}-\frac{1}{18}\mathbf{b}$ or $\overrightarrow{CQ}=\frac{1}{2}\mathbf{a}-\frac{1}{6}\mathbf{b}$ (B1 for one of the above)
	M1	For $\overrightarrow{CQ}=\frac{3}{2}\overrightarrow{CP}$ oe or $\overrightarrow{PQ}=2\overrightarrow{CP}$ oe or $\overrightarrow{CQ}=3\overrightarrow{PQ}$
	A1cso	For a correct conclusion with supporting working

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